



ISTITUTO E MUSEO  
DI STORIA DELLA SCIENZA

# Galileo's compass

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# 1 HISTORY

## 1.1 HISTORY OF AN INVENTION

### The invention

Throughout the Renaissance, many attempts were made to develop a universal instrument that could be used to perform arithmetical calculation and geometric operations easily. This need was felt especially in the military field, where the technology of firearms called for increasingly precise mathematical knowledge. To satisfy these requisites, the first proportional compasses were developed in the second half of the sixteenth century, among them some singular instruments known as the “*radio latino*” and the “*proteo militare*”. The geometric and military compass of Galileo (1564-1642) belonged to this class of instruments. Invented in Padua in 1597, the instrument is also linked to Galileo’s activity in the Accademia Delia, founded in Padua to provide mathematical instruction for young noblemen training for a military career. With the seven proportional lines traced on the legs of the compass and the four scales marked on the quadrant, it was possible to perform with the greatest of ease all sorts of arithmetical and geometric calculations, ranging from calculating interest to extracting square and cube roots, from drawing polygons to calculating areas and volumes, from measuring gauges to surveying a territory. Between 1598 and 1604, Galileo instructed several European sovereigns on the use of his compass, among them Prince John Frederick of Alsace, Archduke Ferdinand of Austria, the Landgrave Philippe of Hesse and the Duke of Mantua.

### The success of the instrument

The success of the instrument encouraged Galileo to divulge his invention still further. In 1606 he published 60 copies of *Le operazioni del compasso geometrico e militare*, each of which he sold privately along with one of the instruments. The production of compasses, from which Galileo earned a substantial profit, was entrusted to an instrument-maker whom the scientist housed for some years in his own home. The publication of the treatise immediately aroused great interest, so intense as to provoke bitter arguments in the academic world over the authorship of the invention. Already in 1607 Baldassarre Capra (c. 1580-1626), one of Galileo’s pupils, tried to claim credit for the invention of the instrument among erudite circles by publishing a treatise in Latin on its operations. Other adversaries of Galileo claimed that the instrument had been invented first by the Dutch mathematician Michel Coignet (1549-1623). Many variations in the instrument were made and, with the addition of new proportional lines, its fields of application were later extended. Specific treatises were written by Michel Coignet, who called it “*compasso pantometro*”, by Muzio Oddi (1569-1639) who called it “*compasso polimetro*”, by Ottavio Revesi Bruti who, adding proportional lines for architectural drawing, called it “*archisesto*”, by Girard Desargues (1591-1661) and other French mathematicians who, adding proportional lines for perspective drawing, called it the “optical or perspective compass”. Numerous variations were developed throughout the seventeenth and eighteenth centuries, while during the course of the nineteenth, the proportional compass was gradually replaced by the dissemination of highly refined slide rules which survived in the technical studios of engineers, architects and geometers up until the very recent advent of the computer.

## THE PICTURES

1. Galileo Galilei, *Le operazioni del compasso geometrico e militare*, ms. 1606, Florence, Biblioteca Nazionale, Ms. Gal. 37, Frontispiece, detail.

The compass of Galileo (1564 – 1642) represents not only an original invention but also the synthesis of numerous attempts made during the course of the Renaissance to develop a

universal instrument that could be used to perform arithmetical calculations and geometric operations easily.

2. **Folding rule of Antonio Bianchini (16<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***

This folding rule, consisting of two wide flat graduated legs, bears various indications, among them the scale of degrees, the quadrant of shadows, the wind rose, a division into equal parts and a list of forty-two Italian cities with their latitudes. It also has a magnetic compass with a scale of degrees next to the hinge point. It was designed to measure position angles and terrestrial and astronomical distances.

3. **Plate portraying soldiers using a compass to measure the barrel of a cannon. Jim Bennet, Stephen Johnston (edited by), *The Geometry of War (1500-1750)*, Oxford, 1996, p. 15**

Using instruments was indispensable in the military field, where the technology of firearms called for increasingly more precise mathematical knowledge.

4. **Mordente's compass (1591), Florence, *Istituto e Museo di Storia della Scienza***

Invented by Fabrizio Mordente (1532 – c. 1608) to measure the smallest fraction of a degree, this particular proportional compass with eight points is distinguished by the presence of sliding cursors. Based on their positions, it was possible to establish the proportions between lines, geometric figures and solid bodies.

5. **Plate illustrating the *radio latino***

**Latino Orsini, *Trattato del radio latino*, Rome, 1583**

The *radio latino* was named for the Roman commander Latino Orsini (ca. 1530 - ca. 1580), its designer. On the four rods jointed to form a parallelogram, which slide along the central axis, are traced various scales of measurement, such as the quadrant of shadows and the scale of degrees. It was used to measure distances, heights and depths, by means of sighting operations.

6. **Plate illustrating the *proteo militare***

**Bartolomeo Romano, *Proteo militare*, Naples, 1595**

Shaped like a dagger, the "*proteo militare*" offered a vast range of functions, from observation to calculation, in a single instrument. As indicated by its name and its dagger-like shape, the instrument was designed for use especially in the military field.

7. **Portrait of Galileo Galilei,**

**Anonymous Tuscan artist, *Galileo Galilei*, Vienna, *Kunsthistorisches Museum***

The invention of the compass is related to the teaching activity of Galileo (1564 – 1642), both public and private. His participation in the educational programs of the Accademia Delia, a new institution established in Padua for the training of young noblemen destined to a military career, was important in this sense.

8. **Plate portraying two gunners, instructed by a third, engaged in measuring cannon bores**

**Jim Bennet, Stephen Johnston (edited by), *The Geometry of War (1500-1750)*, Oxford, 1996, p. 14**

The teaching of the Accademia Delia, with which Galileo was in contact during his stay in Padua, was concentrated not only on equitation and training to bear arms but also on mathematics.

9. **Geometric and military compass of Galileo Galilei (c. 1606), Florence, *Istituto e Museo di Storia della Scienza*.**

The *Istituto e Museo di Storia della Scienza* of Florence possesses one of the very rare

surviving examples of Galileo's compass, probably the one donated by the Pisan scientist to Cosimo II along with a printed copy of the *Operazioni del compasso geometrico et militare*.

**10. Engraving of a geographic chart of Europe (1569)**

**Gerardo Mercatore, Rotterdam, *Maritiem Museum Pris Hendrik***

Between 1598 and 1604 Galileo (1564 – 1642) instructed a number of European sovereigns in using his compass, among them Prince John Frederick of Alsace, Archduke Ferdinand of Austria, Landgrave Philippe of Hesse and the Duke of Mantua, as he himself stated in the Foreword to Readers of his *Le operazioni del compasso geometrico e militare*.

**11. Galileo Galilei, *Le Operazioni del compasso geometrico e militare*, Padua, 1606, Frontispiece**

In 1606, consequent to the success of his instrument, Galileo (1564 – 1642) published 60 copies of *Le operazioni del compasso geometrico e militare*, selling them privately along with the instruments. Dedicated to Prince Cosimo de' Medici (1590 – 1621), future Grand Duke of Tuscany (from 1609 to 1621), the text describes in detail the use of the instrument and the operations that can be performed with it. But the author does not explain how the scales were constructed, perhaps to protect the invention from possible rivals.

**12. Engraving portraying a compass-maker in his shop (1568)**

**Maya Hambly, *Drawing Instruments (1580-1980)*, London, 1988, p. 21**

On July 5, 1599 the mechanic Marc'Antonio Mazzoleni went to live in the home of Galileo (1564 – 1642), engaged to construct the mathematical instruments designed by the scientist, among them the *geometric and military compass*.

**13. Baldassarre Capra, *Usus et fabrica circini cuiusdam proportionis*, Bologna, 1655, Frontispiece**

In 1607 Baldassarre Capra (c. 1580-1626), a pupil of Galileo (1564 – 1642), attempted to claim credit for having invented the compass, publishing a treatise on its operations in Latin entitled *Usus et fabrica circini cuiusdam proportionis* [Use and construction of the proportional compass]. On April 9, Galileo accused Capra of plagiarism. After the trial, in which Capra was found guilty, Galileo published his *Difesa contro alle calunnie et imposture di Baldessar Capra* (Defense against the calumny and false claims of Baldessar Capra).

**14. Drawing of Baldassarre Capra's compass**

**Baldassarre Capra, *Usus et fabrica circini cuiusdam proportionis*, Padua, 1607**

The compass of Baldassarre Capra (c. 1580-1626), like that of Galileo (1564 – 1642), has flat legs marked with proportional lines converging at the center of the joint.

**15. Michel Coignet's "pantometro" compass**

**Michel Coignet, *Usus duodecim divisionem regulae pantometrae*, ms., 1610-1613, Florence, Bibl. Riccardiana, 859**

In 1610, during the anti-Galilean argument over the origin of the telescope, Giovanni Camillo Gloriosi attributed the invention of the proportional compass to the Flemish mathematician and military engineer Michel Coignet (1549-1623). He is instead the inventor of the "pantometro" compass, formed of two flat-legged compasses bearing on each side three proportional scales, designed to solve mathematical, geometric, stereometric, trigonometric, gnomonic and cartographic problems.

**16. Michel Coignet's "regula pantometra"**

**Michel Coignet, *De regulae pantometrae fabrica et usu libri septem*, ms., Oxford, Bodleian Library, Canon Misc. 243**

The invention of the "pantometro" compass by Coignet (1549-1623) probably dates from as early as 1596, when he divulged the "regula pantometra", a proportional compass with flat

legs, mounted on a tripod. Described mainly as a topographic instrument, it was equipped with optical sights and a folding rule with proportional scales marked on it.

**17. Muzio Oddi, *Fabbrica et uso del compasso polimetro di Mutio Oddi da Urbino, Milan, 1633, Frontispiece, detail***

A disciple of Guidobaldo del Monte (1545 – 1607), Muzio Oddi (1569 – 1639) furnished the first important information on the research of the Urbino mathematicians which preceded Galileo's invention.

**18. Plate with drawing of an *archisesto***

**Ottavio Revesi Bruti, *Archisesto per formar con facilità li cinque ordini d'architettura, Vicenza, 1627***

Ottavio Revesi Bruti developed a particular type of proportional compass, the *archisesto*, which had proportional lines for drawing the architectural orders.

**19. Box for mathematical instruments (17<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***

This box contains a set of mathematical instruments dating from the 17<sup>th</sup> century, coming from the Medicean collections. The interior, divided into nineteen compartments, now holds thirteen pieces, all made of brass: various instruments for drawing, a pair of knives and a proportional compass.

**20. Drawing of slide rule with various scales marked on it**

**Peter Nicholson, *Practical Builder, 1837***

During the course of the nineteenth century, the proportional compass was gradually replaced by highly refined slide rules which survived over the years in the technical studios of engineers, architects and geometers up until the recent advent of the computer.

## **1.2 COMPARISON BETWEEN COMPASSES**

The term "compass" or "compasses" denotes a wide range of instruments for drawing, measurement, and proportional calculation. Besides the more common compasses for drawing circumferences, widespread since antiquity, the Renaissance has left us a great number of special compasses, whose names reflect their specific functions: oval compasses to draw ellipses, hyperbolas and parabolas; two-point compasses—also called *dividers*—to divide lines and transfer measurements; nautical compasses, to track routes on sea-charts; compasses with curved points, called gunner's compasses, to measure mouths of cannons, cannon balls, and columns; three-legged compasses to reproduce maps; four-point compasses, or reduction compasses, to enlarge or reduce drawings, divide lines and circumferences proportionally, and draw polygons; eight-point compasses to measure fractions of degrees and carry out proportional calculations; proportional compasses to perform arithmetic, geometric, and trigonometric calculations and to measure weights, gradients, and distances for military use; and sophisticated surveying compasses that combined a magnetic compass, a windrose, and optical sights for surveying and for drawing topographic maps.

## **THE PICTURES**

**1. Plate illustrating various types of compass.**

**Nicolas Bion, *Traité de la construction et des principaux usages des instruments de mathématique, plate VIII, Paris, 1709***

Grouped under the name of compass is a wide range of instruments for drawing, measuring and proportional calculation.

2. **Compass with two points (Pompeii, 1<sup>st</sup> ca. A.D.), Naples, MAN, inv. 76679**  
This compass (*circinus*) was found at Pompeii. Composed of two flat legs, it was widely used in construction sites and in wall decorations.
3. **Leonardo da Vinci 's compass for drawing ellipses, reconstruction (from Benvenuto della Volpaia, *Macchine e strumenti*, Venice, Biblioteca Nazionale Marciana, It. 5363, c. 18r), Florence, *Istituto e Museo di Storia della Scienza***  
This compass, attributed to Leonardo da Vinci (1452 – 1519), served to draw ellipses. The longer leg represented the axis of a cone, while the one with the sliding stylus, adjustable on the graduated arc, represented the generatrix, which was rotated around the axis in such a way that the sliding stylus could trace on a sheet (oblique in respect to the axis of the cone) the figure of an ellipse.
4. **Dividers (16<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
Compass with fixed points used in architectural drawing and on construction sites to trace circumferences, draw right angles and equilateral triangles, take measurements and divide lines into equal parts. It could be either simple or fitted with a micrometric screw for greater precision in the opening of the legs.
5. **Nautical compass (17<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
Its particular conformation allows this compass to be firmly grasped, so that it can be opened and used with one hand only. It was used to measure distances on nautical charts and to trace navigation routes.
6. **Caliper (17<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
Formed of two curved legs terminating in a point, this compass was used to measure the diameters of spherical and cylindrical bodies and the size of cavities. In military practice, it was used to measure the bores of artilleries and the diameters of bullets. In architecture it was used for measuring and fabricating columns. In the field of sculpture it was used to transpose the measurements of a model onto the piece being sculpted. The caliper was often equipped with a graduated arc by means of which measurements could be read directly.
7. **Compass with three legs (17<sup>th</sup> ca.) Florence, *Istituto e Museo di Storia della Scienza***  
The compass with three legs, also called spherical compass or globe compass, was used to transfer onto a chart the positions of places on a terrestrial or celestial globe, by measuring the distance of three points. It could also be used to reproduce a drawing or a geographical chart on the same scale.
8. **Reduction compass (17<sup>th</sup> ca.) Florence, *Istituto e Museo di Storia della Scienza***  
Compass with crossed legs, having a mobile center, whose opposite points form simple ratios of 1:2, 1:3, and so on. The instrument was used essentially to reproduce drawings in a reduced or enlarged scale.
9. **Mordente's compass (late 16<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
This particular proportional compass, invented by Fabrizio Mordente (1532 - ca. 1608) to measure the smallest fraction of a degree, is distinguished by the presence of sliding cursors with eight points orthogonal to the axis of the legs. Based on the position of the points, measured with the aid of a ruler marked with proportional scales, it was possible to establish the proportions between lines, geometric figures and solid bodies.

10. **Proportional compass (17<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
Proportional compass, distinguished by the presence on the legs of various proportional scales which served to establish the proportions between lines, geometric figures, solid bodies, and so on. This particular type of flat-legged proportional compass was perfected by Galileo (1564 - 1642) with his geometric and military compass.
11. **Topographic compass (16<sup>th</sup> ca.), Florence, *Istituto e Museo di Storia della Scienza***  
A topographic compass was generally formed of two flat graduated legs, fitted with sights at the ends and a magnetic compass with the scale of degrees at the hinge point. It was designed to measure the position angles and distances of places in topographic surveying.

### 1.3 THE COMPASS GAME

1. **Caliper – Measure the diameter of a sphere**  
Thanks to the two curved legs terminating in a point it was possible, with this compass, to measure the diameter of spherical bodies, such as the diameter of bullets.
2. **Compass for drawing ellipses – Draw an ellipse**  
With this compass it was possible to draw an ellipse by rotating the leg with the sliding stylus around the longer leg.
3. **Dividers – Draw a circumference**  
This ordinary compass with two fixed points could be used to draw a circumference.
4. **Reduction compass – Reproduce a drawing in scale**  
This compass could be used to reproduce a drawing in scale, in this case in the scale of 1:3. The center of the compass, in fact, is adjusted so that the opposing points form a ratio of 1:3.
5. **Mordente's compass – Measure fractions of degrees**  
This compass perfected the use of astronomical instruments, such as the quadrant or the astrolabe. By positioning the points according to a determined proportion, it was possible to measure with extreme precision each minimum fraction of a degree, such as minutes and seconds.
6. **Compass with three legs – Reproduce a map**  
This compass could be used to reproduce a geographic chart in the same scale, by measuring the distances of three points from one another.
7. **Proportional compass – Carry out arithmetical calculations**  
Thanks to the proportional scales marked on the legs, it was possible to calculate the fourth unknown term of a proportion, making use of the properties of similar triangles.

## 2 EXPLORE

### 2.1 THE INSTRUMENT

#### INFORMATION AND CHARACTERISTICS

Name of instrument: **Geometric and military compass**

Invented and constructed by: **Galileo Galilei**

Date of invention: **1597**

Material: **Brass**

Length: **256 mm**

Width (open): **360 mm**

Thickness: **5 mm**

#### COMPONENTS

##### Arms

Made of brass, with rectangular section, they are hinged in a round disk, called *nocella*, and decorated with engraving according to the fashion of the times. The lower ends are embellished by a worked foot. On the sides, *recta e versa*, are engraved seven proportional scales.

##### Quadrant

An accessory which is fixed, by means of wing nuts, to the holes drilled in the arms of the compass, locking the opening of the instrument at a  $90^\circ$  angle. On the *recto* are engraved four graduated scales.

##### Zanca

A cursor which, inserted in one of the arms of the compass, serves to lengthen it.

##### Plumb line

It is fixed to the *nocella* when the compass is used as *archipenzolo*, or as *astronomical quadrant*.

##### Wing nuts

These screws serve to fix the quadrant to the arms of the compass.

#### LINES

##### CONSTRUCTION AND FUNCTION OF ARITHMETIC LINES

The arithmetic lines are scales divided into 250 equal parts. They are used to perform addition, subtraction, multiplication and division.

##### CONSTRUCTION AND FUNCTION OF GEOMETRIC LINES

The geometric lines are proportional scales which indicate the sides of squares having surface area of 1, 2, 3, respectively, up to a surface area of 50. For this reason the distances of points 1, 2, 3, up to 50, from the center of the compass are equal to 1,  $\sqrt{2}$ ,  $\sqrt{3}$ , up to  $\sqrt{50}$  respectively. These lines are used to carry out geometric operations on plane figures and to extract square roots.

##### CONSTRUCTION AND FUNCTION OF STEREOMETRIC LINES

The stereometric lines are proportional scales which indicate the sides of cubes having volumes of 1, 2, 3 respectively, up to volume 140. For this reason the distances of points 1, 2, 3, up to 140, from the center of the compass are equal to 1,  $\sqrt[3]{2}$ ,  $\sqrt[3]{3}$ , up to  $\sqrt[3]{140}$ . These lines serve to carry out geometric operations on regular bodies and to extract cube roots.

### **CONSTRUCTION AND FUNCTION OF METALLIC LINES**

The metallic lines are proportional scales which indicate the diameters of balls of different metals having the same weight. The metals, indicated by specific weight in decreasing order, are gold, lead, silver, copper, iron, tin, marble and stone. These lines were utilized by artillerymen to calculate the caliber of missiles.

### **CONSTRUCTION AND FUNCTION OF POLYGRAPHIC LINES**

The polygraphic lines are proportional scales which indicate the radiuses of circles circumscribed in regular polygons having the same side, from the triangle to the 15-side polygon. In the case of the hexagon, the radius and the side are equal. Thus the distance identified by point 6 is also equal to the side of all of the other polygons. These lines serve to draw regular polygons and to divide a circle into equal parts.

### **CONSTRUCTION AND FUNCTION OF TETRAGONIC LINES**

The tetragonic lines are proportional scales which indicate the sides of regular polygons having the same area, from the triangle to the polygon with 13 sides. They are used to find the area of regular polygons.

### **CONSTRUCTION OF ADDED LINES**

The added lines, so called because added to the tetragonic lines, are two proportional scales traced on the same line. The outer scale, divided into 20 equal parts, shows the sagittas of 20 circular sectors having the same chord. The inner one shows instead the sides of 20 squares having areas equal to the respective circular sectors. The added lines are used to find the area of circular sectors.

### **CONSTRUCTION OF GUNNERS' SQUARE**

The first scale on the quadrant is the so-called gunners' square, a scale divided into 12 equal parts which was used to measure the elevation of cannons.

### **CONSTRUCTION OF ASTRONOMICAL QUADRANT**

The second scale on the quadrant is the astronomical quadrant, a scale divided into 90 equal parts which was used to measure the height of stars above the horizon.

### **CONSTRUCTION OF SCALE OF SLOPES**

The third scale on the quadrant is the scale of slopes, which shows the inclination of the slope of walls from a ratio of one to one point five between height and escarpment, to a ratio of one to ten.

### **CONSTRUCTION OF SHADOWS SQUARE**

The fourth scale on the quadrant is the shadows square, a scale divided into 200 unequal parts obtained by projecting onto the arc of a circle the divisions into equal parts of the sides of a circumscribed square. This scale was used to measure inaccessible distances.

## **2.2 HOW IT WORKS**

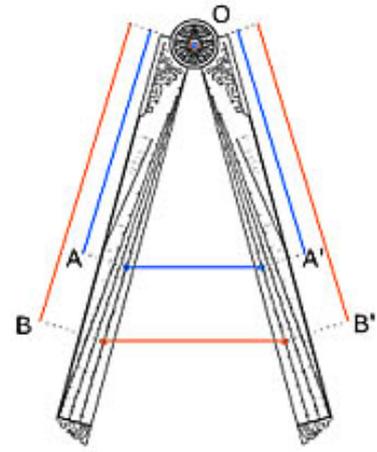
All of the uses of the compass (except when it is used as square or quadrant) derive from the possibility of finding with it quantities proportional to assigned quantities, whether they be lines, areas or volumes.

Consequently, the basic operation of this instrument, as of similar ones, is *proportion*, which can be expressed as the equation  $\mathbf{a:b = c:x}$ , where  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ , are known quantities while  $\mathbf{x}$  is unknown.

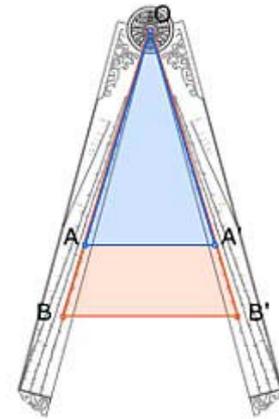
When the compass, in fact, is open at any angle whatsoever, the transverse distances between pairs of matching points are to one another as the distances between those points and the origin.

$$AA':BB'=OA:OB=OA':OB'$$

Why are these segments in proportion?



Let's imagine them as sides of the isosceles triangles  $OAA'$  and  $OBB'$ . These triangles are similar, since the three angles of each are equal. According to **Thales' theorem** on the proportionality of similar triangles, two triangles of different size whose angles are equal, have sides whose length is proportional.



## 3 SIMULATION

### 3.1 THE OPERATIONS

Galileo's compass resembles a calculator. In *Le operazioni del compasso geometrico e militare* (Padua, 1606), Galileo describes over 40 operations that can be carried out with his instrument.

Try using the compass yourself! Choose one of the operations from the list and try to solve some of the problems for which Galileo invented it.

Have fun!

#### 3.1.1 Division of a segment (*RECTO – Arithmetic lines*)

##### Introduction

The arithmetic lines can be used to divide a segment into any number of equal parts.

For this reason Galileo's compass could be used for geometric and architectural drawing, for constructing the graduated scales on measuring instruments and for similar applications.

##### Example

Let's try to divide a segment into 5 equal parts.

On the arithmetic lines, we will find two numbers which have a 5 to 1 ratio, such as 100 and 20.

We will then open the compass so as to apply the measurement of the segment between the two points marked 100.

Keeping the compass at this opening, the distance between the two points marked 20 will be equivalent to one-fifth of the given segment.

By applying this measurement 5 times to the segment, we obtain its exact division into 5 equal parts.

We can obtain the same result by choosing other pairs of number, on condition that the ratio between them is 5:1, as for example: 150-30, 210-42.

##### Try it yourself!

#### INSTRUCTIONS

**Divide the segment into 8 equal parts! Give an exact answer to each of the questions you will be asked, in order to carry out the operation correctly.**

**Good luck!**

**Question 1. To divide a segment into eight parts, which pair of numbers should you use on the arithmetic line?**

1. 100-20.
2. 240-30.
3. 16-71.

##### Solutions:

1. **Wrong!** Because 20 is not one-eighth of 100!

2. **Right!** Because 30 is one-eighth of 240!
3. **Wrong!** Because 16 is not one-eighth of 71!

**Question 2. And now what should you do?**

1. **Measure the line with proportional dividers.**
2. **Set the two compasses at the same opening.**
3. **Measure the distance between points 30 and 240 on the arithmetic lines of the compass.**

**Solutions:**

1. **Right!** The operations of Galileo's compass are carried out with the aid of ordinary proportional dividers with two points, used to take the lengths of the segments involved in the problem so as to apply them to the scales!
2. **Wrong!** The first thing to do is to measure the length of the line. And then setting the compasses at the same opening serves no purpose!
3. **Wrong!** To divide the line it is not necessary to know this distance!

**Question 3. Now you have to set Galileo's compass at the correct opening. How?**

1. **Enter the measurement of the line between the points marked 240 on the two arithmetic lines.**
2. **Enter the measurement of the line between the points marked 30 on the two arithmetic lines.**
3. **At random.**

**Solutions:**

1. **Right!** In this way you have constructed an isosceles triangle whose base measures the same as the line, while the two equal sides measure 240!
2. **Wrong!** Because you have to construct an isosceles triangle whose two equal sides measure 240, and not 30!
3. **Wrong!** The opening of the compass must be adjusted according to the measurement of the line!

**Question 4. How do you find one-eighth of the line?**

1. **Measure the distance between the points marked 80 on the two arithmetic lines.**
2. **Measure the distance between the points marked 240 on the two arithmetic lines.**
3. **By measuring the distance between the points marked 30 on the two arithmetic lines.**

**Solutions:**

1. **Wrong!** This would not give you one-eighth, but one-third of the line!
2. **Wrong!** This distance is the measurement of the line itself!
3. **Right!** Because 30 is one-eighth of 240 and the distance between the points marked 30 is also one-eighth of the distance between the points marked 240!

**Your steps:**

<b>You have chosen 240 and 30 on the arithmetic lines.</b>
<b>You have taken the measurement of the segment to be divided.</b>
<b>You have applied the measurement of the segment between the two points marked 240.</b>
<b>You have measured the distance between the two points marked 30.</b>

**Congratulations! You have just divided a segment into 8 equal parts using Galileo’s compass!**

### **3.1.2 Exchanging currency (*RECTO – Arithmetic lines*)**

#### **Introduction**

Seventeenth-century Italy was a group of many independent states, each of which had its own particular monetary system. Exchanging currency was thus a daily problem for merchants, bankers and even simple travelers.

Often it was necessary to carry out laborious proportional calculations, which could instead be done with Galileo’s compass in a few simple steps, using it as a real “currency converter”.

#### **Example**

Let’s say we want to convert 186 gold *scudi* into Venetian ducats. To do this, the only thing we need to know beforehand, when we don’t know the direct exchange rate of the two currencies, is how much each is worth when exchanged for a third currency, *soldi* for instance.

Knowing that a *scudo* is worth 160 *soldi* and a ducat is worth 124, we set the compass according to these values. To do this we measure on the arithmetic line the value of the *scudi*, which is 160, and we apply it transversely between the pair of points marked 124, which indicate the value of the ducats.

Leaving the compass at this opening, we proceed as follows to solve the problem. To exchange 186 gold *scudi*, we take the transverse distance between the pair of points marked 186 and apply it to the arithmetic line, obtaining 240, which is the value of the ducats.

Proceeding in the same way, we can convert any other quantity of gold *scudi* into Venetian ducats.

Try it yourself!

**INSTRUCTIONS**

Try calculating an exchange rate yourself! Out of the 240 ducats that we had, we have spent 85, leaving us with a remainder of 155. How much money do we still have in *scudi*?

To find the solution, answer the following questions.

Good luck!

**Question 1.** As the first step, open the compass in such a way as to reproduce the proportion between the two currencies to be exchanged. How is this done?

1. Open it so that the distance between the points 124 (the value of a ducat in *soldi*) measures 160 (the value of a *scudo* in *soldi*).
2. Open it so that the distance between the points 160 (the value of a *scudo* in *soldi*) measures 124 (the value of a ducat in *soldi*).
3. Open it at random and measure the value in *soldi* of both the ducat and the *scudo* on the scale of the arithmetic lines.

**Solutions:**

1. **Wrong!** This setting is used to exchange *scudi* for ducats, while you want instead to convert ducats into *scudi*!
2. **Right!** This setting of the compass reproduces the proportion between *scudi* and ducats!
3. **Wrong!** The compass must be set at the correct opening in order to establish the proportion between *scudi* and ducats!

**Question 2.** Having set the compass at the correct opening, where do you measure 155, the value of the ducats to be converted?

1. On the scale of the arithmetic lines.
2. Measure 155 on the scale of the arithmetic lines and apply this length transversely.
3. On the scale of the geometric lines.

**Solutions:**

1. **Right!** The value of the currency to be converted should be applied to the scale of the arithmetic lines!
2. **Wrong!** In this way, 155 would be the value of the *scudi* and not the ducats!
3. **Wrong!** The currency exchange operation is carried out on the arithmetic lines and not the geometric ones!

**Question 3.** As the last step, how do you get the value of a *scudo*?

1. Take the transverse distance between the points marked 155 and measure it on the scale of the arithmetic lines.
2. Measure the segment identified on the scale of the arithmetic lines, by applying the measurement 155 transversely.
3. Take the transverse distance between the points marked 155 and measure it on the scale of the geometric lines.

**Solutions:**

1. **Right!** In this way you have found approximately the number 120, i.e., the value of 155 ducats in *scudi*!

2. **Wrong!** This would give you the conversion of 155 *scudi* into ducats!
3. **Wrong!** This operation is carried out on the arithmetic lines and not the geometric ones!

### Your steps

<b>You have opened the compass, reproducing the proportion between a ducat and a <i>scudo</i>.</b>
<b>You have measured the value of the ducats to be converted on the arithmetic lines.</b>
<b>You have found the corresponding value of the <i>scudo</i> transversely.</b>

**Congratulations! You have successfully exchanged currencies using Galileo's compass.**

### 3.1.3 Arranging armies [*RECTO - Geometric lines*]

#### Introduction

The strong development of the military science of the time, which called for increasingly better mathematical skills, was a powerful stimulus to Galileo in perfecting his compass.

Many of the operations that could be easily solved with Galileo's compass could, in fact, be applied to common military problems. The extraction of a square root, for instance, could serve to solve the problem of deploying an army in square formation.

#### Example

Given the total number of soldiers, for example 3136, the square root could be extracted in a few simple steps with the compass. The resulting number indicated how many soldiers should be placed on each side of the formation, so as to make a compact, impenetrable square.

The first step consists of setting the compass to extract a square root. Let's consider a 4-digit number of which we know the square root, for instance 1600, whose square root is 40. Since squared numbers, that is, areas, are indicated on the geometric line, the number 1600 should be taken on this line. But since this number is too big, we must consider one of its submultiples, for example, the one-hundredth part of it, 16. We then measure 40 on the arithmetic line and we apply that segment between the points marked 16 on the geometric lines.

Leaving the compass at this opening, we can now calculate the square root of 3136. We will consider its one-hundredth part, that is, 31.36, and we will take only the whole number 31, just as we took 16 for 1600. Now we will measure the transverse distance between the points marked 31 on the geometric lines and apply it to the arithmetic scale. The result, 56, is the square root of 3136, that is, the number of soldiers to be deployed on each side of the formation.

Let's try with some other examples, extracting the square roots of 3600 and 2209. You can check to see if the answer is correct by performing this calculation on your computer.

Try it yourself!

**INSTRUCTIONS**

**In a disastrous battle, our army has lost 1615 soldiers. The remaining 1521 soldiers must close ranks as fast as they can. Use the compass and see if you can help them!**

**To find the solution, answer the following questions.**

**Good luck!**

**Question 1 . The number of soldiers to be deployed on each side is found by extracting the square root of 1520. What pair of numbers will you decide to use for setting the compass opening?**

- 1. 2525 – 50.**
- 2. 1600 – 44.**
- 3. 2500 – 50.**

**Solutions:**

- 1. Wrong!** Because 50 is not the square root of 2525!
- 2. Wrong!** Because 44 is not the square root of 1600!
- 3. Right!** Because 50 is the square root of 2500. The opening of the compass should in fact be set utilizing a 4-digit number and its square root, which must be known!

**Question 2. Since 2500 is too big a number, we will divide it by 100, obtaining 25. How do we use 25 and 50 to set the compass?**

- 1. Measure 50 on the geometric lines and apply this distance between the points marked 25 on the geometric lines.**
- 2. Measure 50 on the arithmetic lines and apply this distance between the points marked 25 on the geometric lines.**
- 3. Measure 50 on the geometric lines and apply this distance between the points marked 25 on the arithmetic lines.**

**Solutions:**

- 1. Wrong!** On the geometric lines appear squared numbers, i.e., areas, while 50 is instead a simple number!
- 2. Right!** On the geometric lines appear, in fact, squared numbers, i.e., areas, while on the arithmetic lines appear simple numbers. This opening therefore sets the correct ratio between them!
- 3. Wrong!** On the geometric lines appear squared numbers, while on the arithmetic lines appear simple numbers!

**Question 3. Having established this opening, how do you find the square root of 1521?**

**(Attention! In this case too you will have to divide 1521 by 100 and use the number 15 for the operation)**

- 1. Take the transverse distance between the points marked 15 on the geometric lines and apply it to the arithmetic lines.**
- 2. Take the transverse distance between the points marked 15 on the arithmetic lines and apply it to the geometric lines.**
- 3. Take the transverse distance between the points marked 15 on the geometric lines and apply it to the same geometric lines.**

**Solutions:**

1. **Right!** You have found the square root of 1521, i.e., 39, which is the number of soldiers to be deployed on each side of the formation!
2. **Wrong!** On the arithmetic lines appear simple numbers, while 15 represents, in this case, a squared number!
3. **Wrong!** On the geometric lines appear squared numbers, while we are trying to find a simple number!

**Your steps**

<b>You have chosen the numbers 2500 and 50 at which to set the compass.</b>
<b>You have opened the compass, reproducing the square root ratio.</b>
<b>You have found the square root of 1520.</b>

**Congratulations! You have successfully extracted a square root using Galileo’s compass.**

**3.1.4 Sum of volumes (*RECTO – Stereometric lines*)**

**Introduction**

The stereometric lines are proportional scales which can be used to solve problems of solid bodies, such as the sum of volumes.

On the practical level, this operation was useful – for example – to grain merchants, stonemasons, or builders of walls and fortifications.

**Example**

Let’s imagine that we have three similar solid bodies – three cubes, for instance– and that we want to construct a fourth cube equivalent to the sum of the given volumes.

The compass does not measure the volumes directly. For this reason, we will take the side of the largest cube and open the compass in such a way as to apply this measurement transversely between any two points on the stereometric lines, for example between the pair of points marked 30.

Leaving the compass at this opening, we measure the sides of the other two cubes and apply these measurements transversely to the stereometric lines. We will see that they fall between the pair of points marked 12 and the pair marked 6.

Summing these numbers – 30, 12 and 6 – we obtain 48. The distance between the pair of points marked 48 is the side of the volume we are looking for.

In fact, since the stereometric scales reproduce the proportionality between volumes through the length of their respective sides, the sum calculated is a sum of volumes.

If we adapt the measurements of the three sides to other pairs of numbers, we obtain the same result. Let’s try! Open the compass so as to apply the side of the largest cube between the points marked 25. At this opening, the sides of the other two cubes will fall between the pair of points marked 10 and the pair marked 5. Summing the two numbers, we obtain 40.

We note that the distance between the pair of points marked 40 is equivalent to the measurement found before, because the ratio between the numbers utilized is always the same.

**Try it yourself!**

**INSTRUCTIONS**  
**Try to find the sum of the volumes of 3 spheres!**  
**To guide the operation correctly, answer the questions you will be asked!**  
**Good luck!**

**Question 1. What do you measure first?**

- 1. The volume of each sphere.**
- 2. The circumference of the maximum circle of each sphere.**
- 3. Linear measurements corresponding, for instance, to the diameter of the 3 spheres.**

**Solutions:**

- 1. Wrong!** Galileo's compass cannot be used to work directly on volumes. You have to use linear measurements!
- 2. Wrong!** With the instruments we have, it is impossible to take this measurement!
- 3. Right!** Since the operation in question is a proportion between magnitudes, the result will be correct only if we take magnitudes of the same type!

**Question 2. Where do we apply the measurements of the three diameters?**

- 1. Transversely between pairs of numbers corresponding to the arithmetic lines.**
- 2. Transversely between pairs of numbers corresponding to the stereometric lines.**
- 3. Transversely between pairs of numbers corresponding to the geometric lines.**

**Solutions:**

- 1. Wrong!** Because the arithmetic lines are scales that reproduce the proportional ratio between lines and not between volumes!
- 2. Right!** The stereometric lines reproduce, in fact, the proportional ratio between volumes. This compass opening thus reproduces the proportional ratio between each of the three spheres and the others, whose diameter falls between the stereometric lines of the compass at this opening!
- 3. Wrong!** Because the geometric lines are scales that reproduce the proportional ratio between surfaces and not between volumes!

**Question 3. Now how do we find the diameter of the sphere in question?**

- 1. It corresponds to the transverse distance between the pair of points marked 80 on the stereometric lines ( $80=18+20+42$ ).**
- 2. It corresponds to a segment equivalent to the sum of the 3 diameters.**
- 3. It corresponds to the distance between the center of the compass and the number 80 on the stereometric lines ( $80=18+20+42$ ).**

**Solutions:**

1. **Right!** Since the stereometric lines reproduce the proportional ratio between the volumes, through the length, in this case, of their respective diameters, the sum 80 corresponds to a sum of volumes!
2. **Wrong!** The sum we are looking for is that of the volumes and not of the diameters. In this way we would obtain a sphere with a much larger volume than the one we are looking for!
3. **Wrong!** In this case we would obtain the diameter of a sphere equivalent to the sum of the spheres constructed on the distances 18, 20, 42 of the stereometric lines, not of those constructed on the transverse distances, as in our case!

**Your steps**

<b>You have taken the measurement of the diameters of the 3 spheres.</b>
<b>You have applied the three diameters transversally to the stereometric lines.</b>
<b>You have found the diameter of the sphere whose volume is equivalent to the sum of the three given volumes.</b>

**CONGRATULATIONS! You have successfully found the sum of the volumes of the 3 spheres!**

### 3.1.5 Measurement of calibers (*RECTO - Metallic lines*)

**Introduction**

In Galileo's times, the artillery used without distinction cannonballs made of lead, stone, iron or other materials. Cannonballs made of different materials but having the same caliber (i.e., diameter) have different weights.

Consequently, to standardize the gunfire the gunner had to adjust the explosive charge in relation to the cannonball utilized. The quantity of charge had to be proportionally greater in respect to greater specific weights of the cannonballs.

Galileo's compass could be used as a gunner's gauge, insofar as it established the ratio between the weight and volume of different materials, through the combined use of the metallic lines and stereometric ones.

**Example**

Let's say that we have to bombard a fortification located at a distance of 750 meters. The gunner adjusts the explosive charge for the stone cannonballs and begins to fire.

Finding the stone cannonballs ineffective, the gunner decides to use lead ones, which obviously have a greater specific weight than stone. But how much greater is it? This information is necessary in order to adjust the explosive charge so that the shot is correctly calibrated.

As the first step, we take the diameter of the stone cannonball and apply it transversely between the points marked Pie (that is pietra or stone) on the metallic lines. At this opening we take the distance between the points marked Pio (that is piombo or lead) relevant to lead cannonballs. We thus obtain the diameter of a lead cannonball having the same weight as our stone one.

Thanks to this equivalence, we can now compare the weight of our two cannonballs, utilizing the stereometric lines. We apply the diameter of the lead cannonball, found equivalent in weight to the stone one, between any due points on the stereometric lines, for example between the points marked 10. We then take the diameter of our lead cannonball and see where it falls on the stereometric lines, i.e., between the points marked 65.

From this we also find that the ratio between their weights is 10 to 65, and thus our lead cannonball is 6 and a half times heavier than the stone one. Thanks to this result, the gunner can adjust the explosive charge so as to fire the lead cannonballs at the same force as the stone ones.

**Try it yourself!**

#### **INSTRUCTIONS**

**There are no more lead cannonballs! The gunner must use iron balls of the same caliber, but of different weight! Try to calculate the ratio between the weights of the two cannonballs, in order to regulate the explosive charge!**  
**To guide the operation correctly, answer the questions you will be asked!**  
**Good luck!**

**Question 1. As the first step, set the compass for this operation. How is this done?**

- 1. Open it so as to apply the diameter of the lead cannonball between any pair of points on the stereometric lines.**
- 2. Open it in such a way as to apply the diameter of the lead cannonball between the points marked *Pio* on the metallic lines.**
- 3. Transversely between the points marked *Pie* on the metallic lines.**

**Solutions:**

- 1. Wrong!** The stereometric lines are not used at this point!
- 2. Right!** This opening will let you find the diameter of cannonballs made of different material but having the same weight as our lead cannonball!
- 3. Wrong!** Here we are not interested in stone cannonballs!

**Question 2. This opening will allow you to find the diameter of the iron cannonball equivalent in weight to our lead one. How do you do this?**

- 1. Take the transverse distance between the points marked *Fe* on the metallic lines.**
- 2. Take the transverse distance between the points marked *Pie*.**
- 3. Take the distance between the pivot of the compass and the point marked *Fe*.**

**Solutions:**

- 1. Right!** In this way you have found the equivalent in iron of our lead cannonball and you can then proceed to compare their weights!
- 2. Wrong!** This distance corresponds to the diameter of a cannonball having the same weight as the lead one, but made of stone, not iron!
- 3. Wrong!** This distance corresponds to the diameter of the iron cannonball used as reference for constructing the metallic lines, and not those of our cannonball!

**Question 3. Utilizing the stereometric lines, now compare the weight of our iron cannonball with that of the other one found in the preceding point, equivalent in weight to the lead one. How do you do this?**

- 1. Apply the measurement of the two diameters transversely on the arithmetic lines, verifying between what numbers they fall.**

2. **Apply the measurement of the two diameters transversely on the geometric lines, verifying between what numbers they fall.**
3. **Apply the measurement of the two diameters transversely on the stereometric lines, verifying between what numbers they fall.**

**Solutions:**

1. **Wrong!** The arithmetic lines are used to compare *linear* measurements, while here the lines represent *volumes*!
2. **Wrong!** The geometric lines are used to compare *squared* measurements, while here the lines represent *volumes*!
3. **Right!** The ratio between the numbers found (i.e., between the diameters) is equivalent to the ratio between their volumes and thus between their weights, information necessary to adjust the explosive charge!

**Your steps**

<b>You have opened the compass, applying the diameter of the lead cannonball between the points marked <i>Pio</i> on the metallic lines.</b>
<b>You have found the diameter of the equivalent of the lead cannonball in iron.</b>
<b>You have found the ratio between the weight of the lead cannonball and the iron one.</b>

**Congratulations! You have managed to find the ratio between the weight of a lead cannonball and that of an iron one having the same diameter! With this ratio you can adjust the explosive charge.**

### **3.1.6 Division of a circumference (*VERSO - Polygraphic lines*)**

**Introduction**

The polygraphic lines can be used to divide a circumference into any number of equal parts.

This operation was useful not only for drawing regular polygons, but especially for constructing the graduated scales of measuring instruments of circular shape, such as astrolabes and nocturnal clocks.

**Example**

Let's try to divide a circumference into 10 equal parts. This operation is equivalent to drawing a regular decagon within the circumference.

We will take the measurement of the radius of the circumference. We then open the compass in such a way as to apply it transversely between the pair of points marked 10 on the polygraphic lines (10 like the number of sides of the regular decagon we are looking for).

With this opening, due to the particular construction of the polygraphic lines, the distances between the other pairs of points on the polygraphic lines – for example 8.8, 6.6, 3.3 etc. – will be the radiuses of the circles circumscribed on the regular polygons (in this case an octagon, a hexagon and a triangle) having the same side.

Considering that, among all of the regular polygons, only in the hexagon is the side equal to the radius of the circumscribed circle, the distance between the pair of points marked 6 will be not only the side of the hexagon inscribed in the circle, but also the side of the decagon we are looking for.

**Try it yourself!**

**INSTRUCTIONS**

**Now try it yourself with a nocturnal clock. Divide the circle of the zodiac into 12 equal parts, corresponding to the 12 signs of the zodiac. To find the solution, answer the questions you will be asked. Good luck!**

**Question 1. What do you do first?**

1. **Take the measurement of the radius of the instrument.**
2. **Take the measurement of the circumference of the zodiacal circle.**
3. **Take the measurement of the radius of the zodiacal circle.**

**Solutions:**

1. **Wrong!** We are not trying to divide the outer circle of the instrument, but the zodiacal circle!
2. **Wrong!** With the instruments we have available, we are not able to take this measurement, and then we don't need it!
3. **Right!** This measurement will be one of the terms of the operation that will allow us to find the measurement we are looking for!

**Question 2. Where do you apply this measurement?**

1. **Transversely between any pair of points on the polygraphic lines.**
2. **Transversely between the pair of points marked 12 on the polygraphic lines.**
3. **Transversely between the pair of points marked 6 on the polygraphic lines.**

**Solutions:**

1. **Wrong!** The pair of numbers is not taken at random, but depends on the number of parts into which we want to divide the circumference!
2. **Right!** In this way we establish the ratio between the circumference and the dodecagon that can be inscribed in it!
3. **Wrong!** The pair of points depends on the number of parts into which we want to divide the circumference, which in our case is 12, not 6!

**Question 3. To divide a circle into 12 equal parts, you must find the side of the dodecagon inscribed in the circumference. How can you do this?**

1. **Take the distance between the pair of points marked 6 on the polygraphic lines.**
2. **Take the distance between the pair of points marked 6 on the geometric lines.**
3. **Take the distance between the pair of points marked 10 on the polygraphic lines.**

**Solutions:**

1. **Right!** This measurement corresponds to the radius of the circle circumscribed on the hexagon whose side is the same as that of the dodecagon you are looking for. Since in the hexagon the radius and the side are equal, this measurement is also equivalent to the side of the dodecagon!
2. **Wrong!** Although utilized for surfaces, the geometric lines do not help us to solve our problem!
3. **Wrong!** This measurement does not help us to solve our problem!

**Your steps**

<b>You have taken the measurement of the radius of the zodiacal circle.</b>
<b>You have applied the measurement of the radius between points 12.12 on the polygraphic lines.</b>
<b>You have found the side of the dodecagon by means of which the circle can be divided into 12 equal parts.</b>

**Congratulations! You have managed to divide the zodiacal circle on the nocturnal clock with Galileo’s compass!**

**3.1.7 Squaring of regular surfaces (*VERSO - Tetragonic lines*)**

**Introduction**

The tetragonic lines of the compass are used to perform “squaring” of regular surfaces, i.e., measuring their surface area, or finding the squared area equivalent to that of the given figure.

With the sole aid of the compass and without recurring to complicated arithmetical calculations, it was thus possible to determine the areas of figures consisting of regular polygons or circumferences.

**Example**

Let’s try to perform the “squaring” of a circular surface.

Take the measurement of the radius of a circle. Then open the compass so as to apply this measurement transversely between the two dots marked (O) on the tetragonic lines. This opening of the compass reproduces the ratio between our circle and the ideal one on the basis of which the tetragonic scale has been constructed.

Leave the compass at this opening and take the distance between the pair of points marked 4. This measurement represents the side of the square having a surface area equivalent to that of the given circle. Now it is sufficient to multiply the side by itself in order to “square” the circle.

It is just as simple to determine the side of a regular polygon having an area equivalent to that of the circle. Leaving the compass at the same opening and taking the distances between the pairs of points marked 3, 6, 10, we find, in fact, the sides of the triangle, the hexagon and the decagon having a surface area equivalent to that of the given circle.

Try it yourself!

**INSTRUCTIONS**

**Try to calculate the area of a dodecagon, by “squaring” it.  
To find the solution, answer the questions you will be asked.  
Good luck!**

**Question 1. What does “squaring” a dodecagon mean?**

- 1. Finding a square having a surface area equivalent to that of the given dodecagon.**
- 2. Finding a square that circumscribes the given dodecagon.**
- 3. Finding a square having a perimeter equal to the perimeter of the dodecagon.**

**Solutions:**

- 1. Right!** This equivalence lets you calculate the surface area of the dodecagon, by calculating that of the equivalent square!
- 2. Wrong!** A square constructed in this way would have an area greater than that of the dodecagon!
- 3. Wrong!** “Squaring” refers to surfaces and not to linear measurements!

**Question 2. As the first step, we take the measurement of the side of the dodecagon. Where do we apply it?**

- 1. Transversely between the points marked 4.4 on the tetragonic lines.**
- 2. Transversely between the points marked 10.10 on the tetragonic lines.**
- 3. Transversely between the points marked 12.12 on the tetragonic lines.**

**Solutions:**

- 1. Wrong!** This would be correct in the case of a square, i.e., a regular figure having 4 sides!
- 2. Wrong!** This would be correct in the case of a decagon, that is, a regular polygon having 10 sides!
- 3. Right!** Because a dodecagon is a regular polygon having 12 sides. The opening of the compass obtained in this way reproduces the ratio between our dodecagon and the ideal one on the basis of which the tetragonic scales have been constructed!

**Question 3. How do you find the side of the square you are looking for?**

- 1. Leaving the compass at this opening, I take the distance between the points marked 4.**
- 2. Leaving the compass at this opening, I take the distance between the points marked 8.**
- 3. I take the distance between the origin and the point marked 4 on the tetragonic lines.**

**Solutions:**

- 1. Right!** Thanks to the proportional ratio established by the opening of the compass, this distance represents the side of a square having the same area as that of the dodecagon!
- 2. Wrong!** This distance corresponds to the side of the octagon, not the square, equivalent to that of the dodecagon!
- 3. Wrong!** This distance corresponds to the side of the ideal square on the basis of which the tetragonic scales of the compass have been constructed!

## Your steps

**“Squaring” a dodecagon means finding a square that has the same surface area.**

**Open the compass in such a way as to reproduce the ratio between our dodecagon and the ideal one represented by the tetragonic lines.**

**Find the side of the square by taking the distance between the points marked 4.4 on the tetragonic lines.**

**Congratulations! You have just “squared” a dodecagon using Galileo’s compass.**

### 3.1.8 Squaring sectors of a circle (*VERSO - Added lines*)

#### Introduction

The added lines are used to “square” sectors of a circle, that is, to measure their area by finding a square having the same area.

This operation was especially useful in calculating the area of irregular surfaces, in geometry, in architecture and in the military sphere.

#### Example

Let’s try to calculate the area of a sector of a circle.

With the dividers we take half of the chord and apply this measurement between the points marked D on the added lines.

Leaving the compass at this opening, take the measurement of the arrow of the sector and apply it to the outer scale of the added lines, whose subdivision indicates the arrow of 20 sectors of a circle having the same chord. We will see that this measurement falls between the pair of points marked 10.

On the inner scale are indicated the sides of the squares having surface areas equal to the respective sectors of a circle; consequently, the distance between the points marked 10 on the inner scale corresponds to the side of a square having the same surface area as the sector of a circle.

In similar manner, if the arrow of the sector of a circle had fallen between the points marked 5 on the outer scale, the distance between the points marked 5 on the inner scale would have been the side of the square to be found.

#### Try it yourself!

#### INSTRUCTIONS

**Try to calculate the area of a sector of a circle by “squaring” it.**

**To find the solution, answer the questions you will be asked.**

**Good luck!**

**Question 1. What does “squaring” a sector of a circle mean?**

1. Finding a square having an area the same as that of the given sector of a circle.
2. Finding a square that circumscribes a given sector of a circle.
3. Finding a square having a perimeter the same as the perimeter of the sector of a circle.

**Solutions:**

1. **Right!** This equivalence allows us to find the area of the sector of a circle, by calculating that of the equivalent square!
2. **Wrong!** A square constructed in this way would have an area larger than that of the sector of a circle!
3. **Wrong!** “Squaring” refers to surface areas and not to linear measurements!

**Question 2. As a first step, take the measurement of the semi-chord of the sector of a circle. Where do you apply it?**

1. Transversely between points 4 on the outer scale of the added lines.
2. Transversely between any two points on the added lines.
3. Transversely between the points marked D on the added lines.

**Solutions:**

1. **Wrong!** Between the numbered points on the outer scale of the added lines, the arrow of the sectors of a circle should be applied!
2. **Wrong!** This distance should be applied between a specific pair of points, in order to establish the correct opening of the compass!
3. **Right!** In this way we establish the ratio, in respect to the chord, between our sector of a circle and the one on the basis of which the added scales have been constructed!

**Question 3. In addition to the chord, the other information necessary to identify the sector of a circle is its arrow. Where do you apply it?**

1. I see where it falls transversely, between a pair of points on the *outer* scale of the added lines.
2. I see where it falls transversely, between a pair of points on the *inner* scale.
3. I see where it falls transversely, between a pair of points on the tetragonic lines.

**Solutions:**

1. **Right!** The outer scale, in fact, has been constructed by applying the different arrows of sectors of a circle having the same chord!
2. **Wrong!** The inner scale shows the sides of squares having area equal to that of the sector of a circle constructed on the outer scale!
3. **Wrong!** The tetragonic lines are used to perform squaring of regular figures and not sectors of a circle!

**Question 4. Lastly, how can you find the side of a square with area equivalent to the sector of a circle?**

1. Take the distance between points 4 on the *inner* scale of the added lines.
2. Take the distance between points 8 on the tetragonic lines.
3. Take the distance between points 8 on the *inner* scale of the added lines.

**Solutions:**

1. **Wrong!** This would be right if the arrow of the sector of a circle had fallen between points 4 on the outer scale of the added lines!

2. **Wrong!** The tetragonic lines are used to perform squaring of regular figures and not sectors of a circles!
3. **Right!** Due to the particular construction of the added lines, this distance corresponds to the side of a square with surface area equivalent to that of the sector of a circle!

### Your steps

<b>“Squaring” the sector of a circle means calculating its area by finding a square having the same area.</b>
<b>You have applied the semi-chord of the sector of a circle to the compass, setting it at the correct opening for this operation.</b>
<b>You have applied the arrow of the sector of the same circle between points 8 on the outer scale of the added lines.</b>
<b>You have taken the distance between points 8 on the inner scale of the added lines, i.e., the side of the square equivalent to the given sector.</b>

**Congratulations, you have managed to square a sector of a circle with Galileo’s compass!**

### 3.1.9 Gunners’ square (*QUADRANT- 1<sup>st</sup> Scale*)

#### Introduction

Used in combination with the quadrant and a plumb line, Galileo’s compass could be utilized as a gunners’ square, that is, to measure the elevation of cannons.

#### Example

To use it precisely for this purpose, the leg is inserted into the muzzle. The plumb line, perpendicular to the ground, registers on the first scale of the quadrant the inclination of the inserted arm, and thus of the cannon.

This procedure, although correct, had the defect of exposing the gunner to enemy fire. To avoid this risk Galileo designed an accessory, the so-called *zanca*, which allowed the elevation of a cannon to be measured while keeping the compass on the back of the cannon near the breech. Given the variable thickness of the cannon, the *zanca* served to position the compass parallel to the cannon bore.

### 3.1.10 Astronomical quadrant (*QUADRANT- 2<sup>nd</sup> Scale*)

#### Introduction

Used in combination with the quadrant and a plumb line, Galileo's compass could be utilized to perform astronomical operations, necessary above all for orientation during navigation on the open seas.

The second scale of the quadrant, the astronomical quadrant, could be used, in fact, to measure the declination of the Sun, the height of a heavenly body, the distance between two stars and other astronomical operations.

#### Example

Let's try to measure the height of a star above the horizon. First of all, we will position the compass so that its arm is sighting the star.

Then, keeping the compass at this inclination, we will observe where the plumb line falls in respect to the second scale of the quadrant. We see that the plumb line indicates point 32. The star is thus positioned at 32° above the horizon.

### 3.1.11 Scale of slopes (*QUADRANT- 3<sup>rd</sup> Scale*)

#### Introduction

By applying the quadrant and the plumb line to the small hole at the end of the gunners' square, Galileo's compass could be utilized to measure slopes, such as the slope of a wall.

#### Example

Lets try to measure the inclination of the walls of a fortress. We will rest the arm of the compass against the wall and check where the plumb line falls in respect to the scale of slopes, for example at point 3.

This indicates the ratio existing between the height of the wall and its escarp, in this case 3 to 1. This ratio measures the slope of the wall itself. The higher the ratio, the greater is its slope.

### 3.1.12 Shadow square (*QUADRANT- 4<sup>th</sup> Scale*)

#### Introduction

The last scale on the quadrant is the shadow square, which was used to measure inaccessible distances, such as the height of towers or of mountains, the depth of wells, or the distance between two far-off places.

#### Example

Let's try to measure the height of a tower. First of all, the compass is mounted vertically on a stand, if possible at a distance of 100 steps from the tower.

We then sight the top of the tower. If the plumb line falls in the half of the quadrant opposite the surveyor's eye, the number indicated, for example 92, corresponds to the height in question, that is to say 92 steps.

On the quadrant will appear, in fact, a triangle proportional to the one having as cathetuses the height of the tower and the distance from which it is observed, so it can be said that the height in question  $x$  is to the distance of 100 steps, as 92 is to 100 on the quadrant.

If instead the plumb line falls in the half closest to the eye of the observer, let's say at point 81, we will have the following proportion: the height of the tower  $x$  is to the distance of 100 steps, as 100 is to 81 on the quadrant. In solving the proportion, which imposes equality between the product of the averages and that of the extremes, we find that the height in question is equal to 10,000 divided by 81, or 123 steps. When the plumb line falls in the half nearest to the eye, then, it is sufficient to divide 10,000 by the number indicated.

### **THE TEXT**

In June of 1606 Galileo had printed at his home in Padua, by the printer Pietro Marinelli, 60 copies of *Le operazioni del compasso geometrico e militare*. Each booklet was sold along with one of the instruments whose numerous applications it described. Perhaps for the purpose of preventing any plagiarism of his invention, Galileo failed to explain how the proportional lines were constructed.

The text is prefaced by a *Dedica (Dedication)* to the young Prince Cosimo de' Medici (1590-1621), whom Galileo personally instructed in the use of the compass, and by a *Proemio ai Lettori (Preface to Readers)* in which the scientist revealed important details of his invention.

To read the whole Galileo's text or download it onto your computer go to RESOURCES section of the web site.